

Causal Analysis in Theory and Practice

On the Causal Hierarchy and Robins and Richardson's MCM

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Judea Pearl writes:

Thomas' latest posting triggered my curiosity to re-examine the causal hierarchy. Originally (see Causality chapter 1), I have characterized causal sentences into three categories:

1. probabilistic (i.e., non-causal, or what we can estimate from observational studies)
2. experimental (i.e., do-expressions, or what we can estimate from controlled, randomized experiments)
3. counterfactuals (i.e., subscripted sentences, or everything that can be computed from a fully specified structural model that is, a collection of functions with probabilities on the exogenous variables) ¹

This three-layer classification constitutes a hierarchy, in the sense that sentences at level i are special cases of sentences in level $i+1$ but not the other way around. For example, probabilistic sentences are special cases of do-expressions, in which the do operator does not appear. Likewise do-expressions are special cases of counterfactual sentences in which all subscripts are the same. e.g., $P(Y=y|do(X=x),Z=z) = P(Y_x = y | Z_x=z)$ ²

Where is R&R's MCM in this hierarchy?

R&R posit a new kind of DAGs, defined by Minimal Causal Model (MCM), as an alternative to the DAGs used in the experiments and counterfactual layers. The latter, known as path diagrams, or functional graphs, are labeled NPSEM in R&R's paper. The former, usually known as "manipulation graphs" "intervention graphs", or "Causal Bayesian Networks" (CBN) were labeled "Agnostic DAGs" by R&R. (These were defined on page 24 of Causality in terms of invariance of certain conditional probabilities under certain interventions)

¹ Robins and Richardson (R&R) named level 2 "manipulable contrasts" and describe it as: a set of all contrast between treatment regimes in an experiment with sequential treatment assignments, wherein the treatment given at stage m is the function of past covariates [on the graph].(R&R page 18) These contrasts were named "conditional actions" in Causality ch. 4, and are expressible in do-calculus

² Note that the expression $P(Y=y|do(X=x),Z=z)$ also applies to observations $Z=z$ that occur before the action $do(X=x)$. This is because the action $do(X=x)$ does not affect the past, which implies $Z_x=Z$. Note also that knowledge of temporal precedence, e.g., that $Z=z$ occurs before the action, or its unit-level ramification $Z_x=Z$, cannot be expressed as separate sentences in do-calculus; for they involve equality of counterfactuals at different worlds.

What is an MCM model?

R&R describe MCM as the set of contrasts derivable from intervention distributions, the laws of probability and the consistency rule. Thus, MCM can be viewed either as a hybrid of all three layers in the causal hierarchy, or as a restriction on the counterfactual layer, where the only non-manipulative counterfactual allowed is the consistency rule (and its derivatives).

Readers familiar with do-calculus may wonder how the consistency rule can be applied to do-statements to derive new statements. This is done in two steps, first translate each do-statement into its counterfactual image, then apply the consistency axiom as an inference rule. The former step is always feasible, since the do-calculus is a subset of the counterfactual language

The philosophical question that rises is whether MCM deserves a special status in the hierarchy, given that, theoretically, there are infinitely many ways of creating hybrid layers. One can choose for example to mix do-statements with any subset of counterfactual statements, say, those involving a maximum of two different worlds (i.e., two different subscripts) or those defensible by strongly held theories (e.g., monotonic functions, temporal precedence, randomization). RR would say that the consistency rule, though untestable, is unique in that it is self evident and simply expresses the existence of counterfactuals without making any specific counterfactual claim.

Critics of MCM would argue that there are many self evident statements in science, not less evident than the consistency rule, yet they are excluded unfairly by MCM. One such statement is "strong ignorability under randomization" (SIUR) namely, that the coin we are using to conduct a randomized trial is independent of all factors that may affect outcome, including both, those that operate under treatments as well as those that operate under control. Formally, $X \perp\!\!\!\perp \{Y_1, Y_0\}$ MCM permits only weak ignorability, even under randomization, because, empirically, one can only test the equality:

$$P(y|do(x)) = P(y|x) \text{ for } x=0, 1$$

which implies

$X \perp\!\!\!\perp Y_1$ and $X \perp\!\!\!\perp Y_0$, not strong ignorability.

It is easy to conceive of a non randomized experiments that would appear to be randomized by satisfying the equalities above and, to prevent such impostors from ever contaminating our conclusions, MCM is prepared to deprive us of valuable conclusions in those cases where the randomization is genuine.

For example, suppose the bounds that R&R derive for the (natural/pure) direct effect can be narrowed substantially by adding strong ignorability onto MCM (I did not check whether this in fact is the case), we would then be deprived of this information even in cases where our experiment is carefully randomized.

This deprivation is of real practical concern because, thus far, MCM has identified only one counterfactual contrast beyond those derivable in do-calculus -- the binary ETT; a quantity that can be identified by ordinary methods, for example, by asking whether ETT is identifiable from interventional and observational distributions absent any causal assumptions (see Shpitser and Pearl 2008 for exhaustive analysis of ETT).

My initial reaction to R&R posting was to congratulate them on the discovery of a new layer in the causal hierarchy. I am now skeptical on two counts:

1. It is too early to judge how rich this new layer is
2. It is not clear whether it is useful to give the MCM layer special privileged status over the many other layers one can synthesize, each defined by allowing a different subset of "defensible" or "self-evident" counterfactual assumptions to enrich the do-calculus.

I still congratulate R&R on their insight into the potential flexibility of the causal hierarchy, though I maintain my preference for unleashing the full power of counterfactual analysis and the vivid representation of structural equation graphs, whenever the knowledge articulated is defensible on scientific grounds.

=====Judea Pearl